3 PRACTICE PAPER

SECTION A

Question numbers 1 to 4 carry 1 mark each.

Q.1 If A is a skew-symmetric matrix of order 3, write the value of |A|.

Q.2 For what value of 'k' is the function $f(x)=\begin{cases}\dfrac{\sin 5x}{3x}+\cos x, & if\ x\neq 0\\ k, & if\ x=0\end{cases}$ continuous at $x = 0$?

Q.3 If $\int_0^a \frac{dx}{4+x^2}=\frac{\pi}{8}$, find the value of a.

OR

Evaluate : $\int x^2 e^{-x^3}dx$

Q.4 Write the distance of the point (3,-5,12) from X-axis.

SECTION B

Question numbers 5 to 12 carry 2 marks each.

Q.5 If A and B are square matrices of order 3 such that $|A|=4, |B|=3$, then find the value of $|2AB|$.

Q.6 The radius r of right circular cylinder is decreasing at the rate of 3cm/min and its height h is increasing at the rate of 2cm/min. When $r = 7$cm and h=2cm, find the rate of change of the volume of cylinder. [Use $\pi=\frac{22}{7}$].

OR

Using differentials, find an approximate value of $\sqrt{37}$.

Q.7 Find $\frac{dy}{dx}$ at $t=\frac{2\pi}{3}$ when $x = 10(t - \sin t)$ and $y = 12(1 - \cos t)$.

OR

If $xy + x = 2 \quad (x \neq 0, y \neq -1)$, verify that $\frac{dy}{dx}.\frac{dx}{dy} = 1$

Q.8 Find the condition that the curves $2x = y^2$ and $2xy = k$ intersect orthogonally.

Q.9 A line passes through the point with position vector $2\hat{i} - 3\hat{j} + 4\hat{k}$ and is perpendicular to the plane $\vec{r}.(3\hat{i} + 4\hat{j} - 5\hat{k}) = 7$. Find the equation of the line in cartesian and vector forms.

Q.10 If $P(A) = 0.4$, $P(B) = p$, $P(A \cup B) = 0.6$ and A and B are given to be independent events, find the value of 'p'.

Q.11 A company produces two types of goods A and B, that require gold and silver. Each unit of type A requires 3 g of silver and 1 g of gold while that of type B requires 1 g of silver and 2 g of gold. The company can produce a maximum of 9 g of silver and 8g of gold. If each unit of type A brings a profit of ₹ 40 and that of type B ₹ 50, formulate a LPP to maximize profit.

Q.12 Evaluate : $\int \frac{dx}{\sin x \cos^3 x}$

OR

Evaluate : $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$

SECTION C

Question numbers 13 to 23 carry 4 marks each.

Q.13 Solve : $\cos^{-1} \frac{x \quad 1}{x^2 + 1} + \tan^{-1} \frac{2x}{x^2 - 1} = \frac{2}{\quad} \; (0 < \quad < 1)$.

Q.14 Using properties of determinants show that

$$\begin{vmatrix} 1 & 1 & 1+x \\ 1 & 1+y & 1 \\ 1+z & 1 & 1 \end{vmatrix} = -(xyz + yz + zx + xy)$$

OR

Find matrix X so that X $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$

Q.15 If $xy = e^{(x-y)}$, then show that $\frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$.

OR

If $\log y = \tan^{-1} x$, then show that $(1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$.

Q.16 Evaluate: $\int \sqrt{\frac{x}{a^3 - x^3}}\,dx$

Q.17 Evaluate: $\int_{-2}^{1} \left|x^3 - x\right| dx$

OR

Find : $\int e^{2x} \sin(3x+1)dx$

Q.18 Find the particular solution of the differential equation $2ye^{x/y}dx + (y - 2xe^{x/y})dy = 0$, , given that $x = 0$ when $y = 1$.

Q.19 Find the area of parallelogram ABCD whose side AB and the diagonal AC are given by the vectors $3\hat{i} + \hat{j} + 4\hat{k}$ and $4\hat{i} + 5\hat{k}$ respectively.

Q.20 If $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}, \vec{b} = 4\hat{i} - 7\hat{j} + \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 6$.

Q.21 There are 4 cards numbered 1 to 4, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean and variance of X.

Q.22 In a shop X, 30 tins of pure ghee and 40 tins of adulterated ghee which look alike, are kept for sale while in shop Y, similar 50 tins of pure ghee and 60 tins of adulterated ghee are there. One tin of ghee is purchased from one of the randomly selected shops and is found to be adulterated. Find the probability that it is purchased from shop Y. What measures should be taken to stop adulteration?

Q.23 Solve the following LPP graphically:

Maximize $Z = 1000x + 600y$

subject to the constraints

$x + y \leq 200,\ x \geq 20,\ y - 4x \geq 0,\ x \geq 0,\ y \geq 0$

SECTION D

Question numbers 24 to 29 carry 6 marks each.

Q.24 If $A=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$ are two square matrices, find AB and hence solve the system of linear equations: $x - y = 3$, $2x + 3y + 4z = 17$ and $y + 2z = 7$.

Q.25 Let $A=\mathbb{R}-\{3\}, B=\mathbb{R}-\{1\}$. Let $f: A \to B$ be defined by $f(x)=\dfrac{x-2}{x-3},\ \forall x \in A$

Show that f is bijective. Also find,

(i) $x,\ if\ f^{-1}(x)=4$ (ii) $f^{-1}(7)$

OR

Let $A=\mathbb{R}\times\mathbb{R}$ and let * be a binary operation on A defined by

$(a,b) * (c,d) = (ad + bc, bd)$ for all $(a,b), (c, d) \in A$

(*i*) Show that * is commutative on A.

(*ii*) Show that * is associative on A.

(*iii*) Find the identity element w.r.t. * in A.

Q.26 A wire of length 34m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a rectangle whose length is twice its breadth. What should be the lengths of the two pieces, so that the combined area of the square and the rectangle is minimum?

Q.27 Using the method of integration, find the area of the triangle ABC, co-ordinates of whose vertices are A(1,2), B(4,3) and C(2,0).

OR

Using integration, find the area of the region $\{(x, y) : x^2 + y^2 \le 1 \le x + y\}$

Q.28 Find the particular solution of the differential equation $\dfrac{dy}{dx}-3y\cot x=\sin 2x$, given that $y = 2$ when $x=\dfrac{\pi}{2}$.

Q.29 Find the vector equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$. Hence, find whether the plane thus obtained contains the line $\dfrac{x+2}{5}=\dfrac{y-3}{4}=\dfrac{z}{5}$ or not.

OR

Find the image P' of the point P having position vector $\hat{i} + 3\hat{j} + 4\hat{k}$ in the plane $\vec{r}.(2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$. Hence find the length of PP'.